National Aeronautics and Space Administration

Global Optimization of Low-Thrust Interplanetary Trajectories Subject to Operational Constraints

Jacob Englander

Matthew Vavrina

David Hinckley

2-16-2016

code 595

NAVIGATION & MISSION DESIGN BRANCH

NASA GSFC

Introduction to Low-Thrust Interplanetary Design

- Low-thrust electric propulsion provides many advantages for mission to difficult targets
  - Comets and asteroids
  - Mercury
  - Outer planets (with sufficient power supply)
- Low-thrust electric propulsion is characterized by high power requirements but also very high specific impulse (I_{sp}), leading to very good mass fractions
- Low-thrust trajectory design is a very different process from chemical trajectory design
  - Like chemical design, must find the optimal launch date, flight time, and dates of each flyby (if applicable)
  - Unlike chemical design, must find a time-history of thrust control for the entire mission
- It is desirable to automate the low-thrust design process as much as possible
- Computer time is CHEAP. Analyst time is EXPENSIVE

Operational Constraints

- A *realistic* preliminary design model for low-thrust trajectories must include:
  - Accurate propulsion, power, and launch vehicle modeling
  - Accurate modeling of operational constraints, *e.g.*,
    - Communication line-of-sight at critical events
    - Solar distance (don’t get too hot)
    - Atmosphere interface and landing targeting
- If operational constraints and spacecraft performance are not accurately modeled, the preliminary design trajectory will not sufficiently resemble the final design and no decisions or trades can be made based on the preliminary design.

Operational Constraints, Continued

- A traditional method is to solve the unconstrained problem many times and see if any of the solutions naturally satisfy the constraints. This can lead to two problems.
  - **False Negatives**: None of the unconstrained solutions naturally satisfy the operational constraints, but viable constrained solutions exist very near the unconstrained solution such that a constrained method would have found them. The analyst may be led to believe that no solutions exist when in fact they do.
  - **False Positives**: There are many good unconstrained solutions, suggesting to the analyst that the problem is tractable. However once operational constraints are applied the problem changes drastically and no viable solutions exist.

Constrained Global Optimization of Low-Thrust Trajectories – a (very) partial bibliography

Global Optimization

Petropolous and Longuski (2004)
Wall and Conway (2009)
Chilan and Conway (2010)
Novak and Vasile (2010)
Wall and Novak (2011)
Taheri and Abdelkhalik (2012)

Constrained Optimization

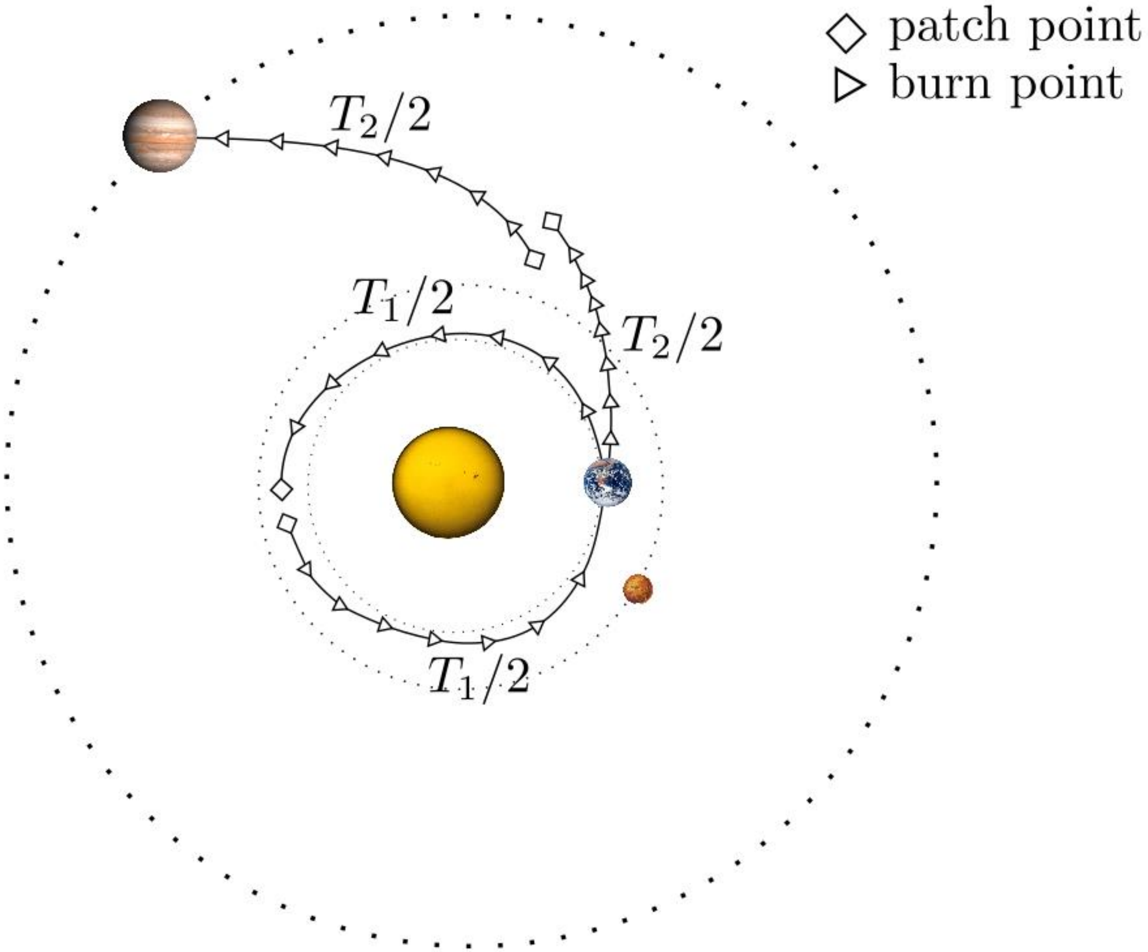
Hargraves and Paris (1987)
Enright and Conway (1992)
Sims and Flanagan (1999)
Benson, Huntington, Thorvaldsen, and Rao (2006)
Ocampo (2010)

Constrained Global Optimization

Woo, Coverstone, and Cupples (2006)
Vavrina and Howell (2008)
Yam, di Lorenzo, and Izzo (2010)
Englander and Conway (2012)
(several other works by these authors)

MODELING

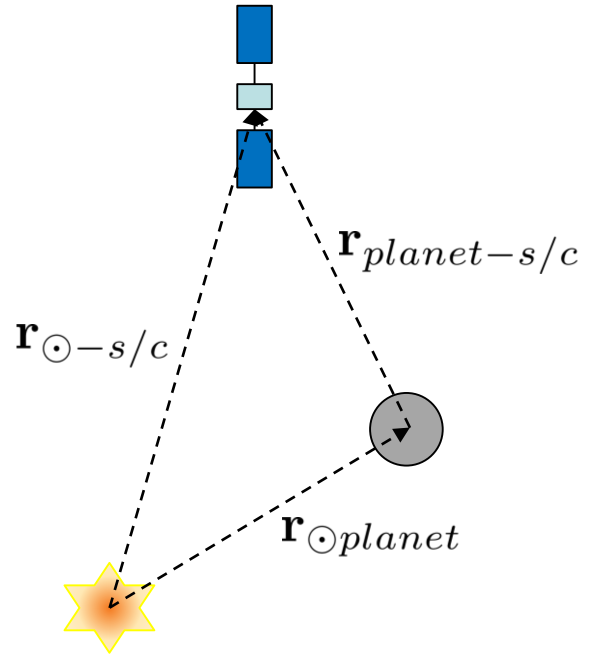
Multiple Gravity Assist with Low-Thrust (MGALT) via the Sims-Flanagan Transcription



Solar Distance Constraint

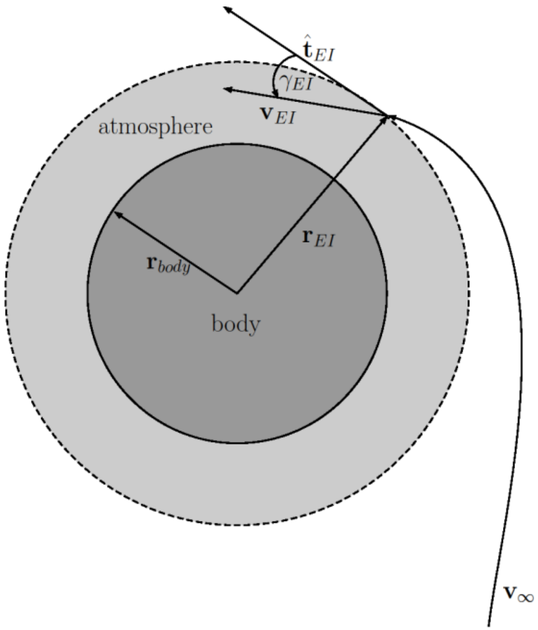
It is often necessary to constraint the spacecraft’s maximum and/or minimum distance from a body in the solar system, *e.g.*

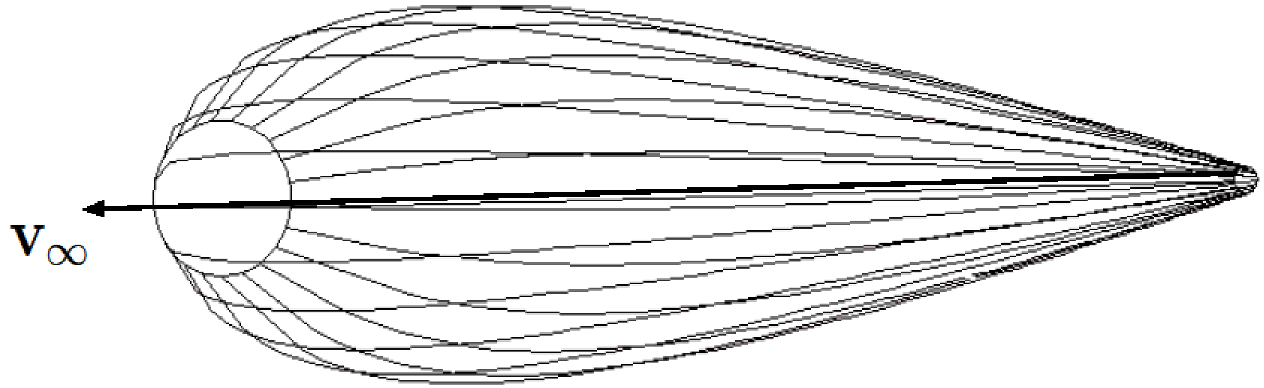
- **Distance from the Sun**: The spacecraft may have thermal constraints such that it cannot survive too close or too far from the Sun.
- **Distance from the Earth**: The spacecraft may have communication constraints such that it not travel too far from the Earth.
- **Death ray on Mars**: You never know what the Martians may be up to…

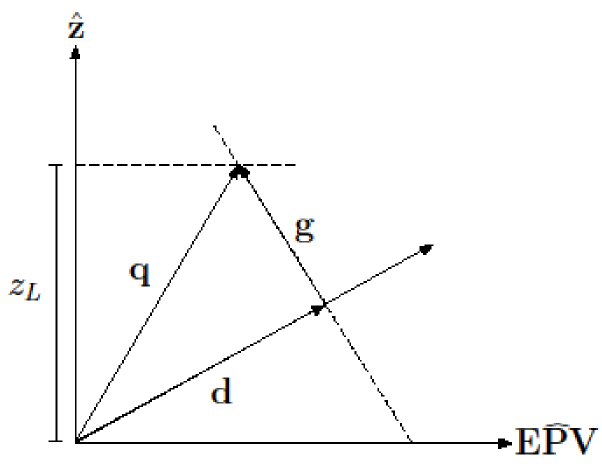


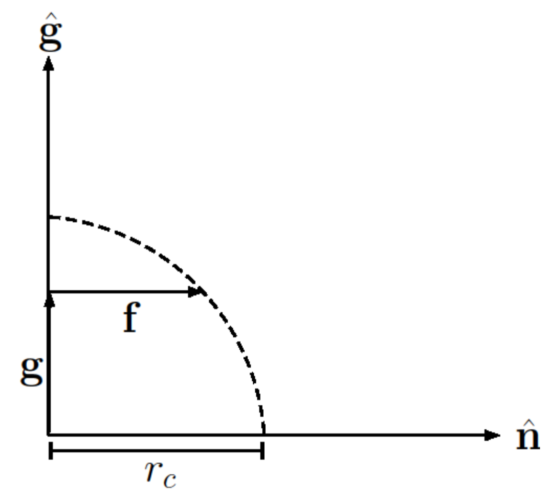
$$d_{LB} \leq r_{body-s/c} \leq d_{UB}$$

Atmosphere Interface/Landing Constraint









Spacecraft Propulsion

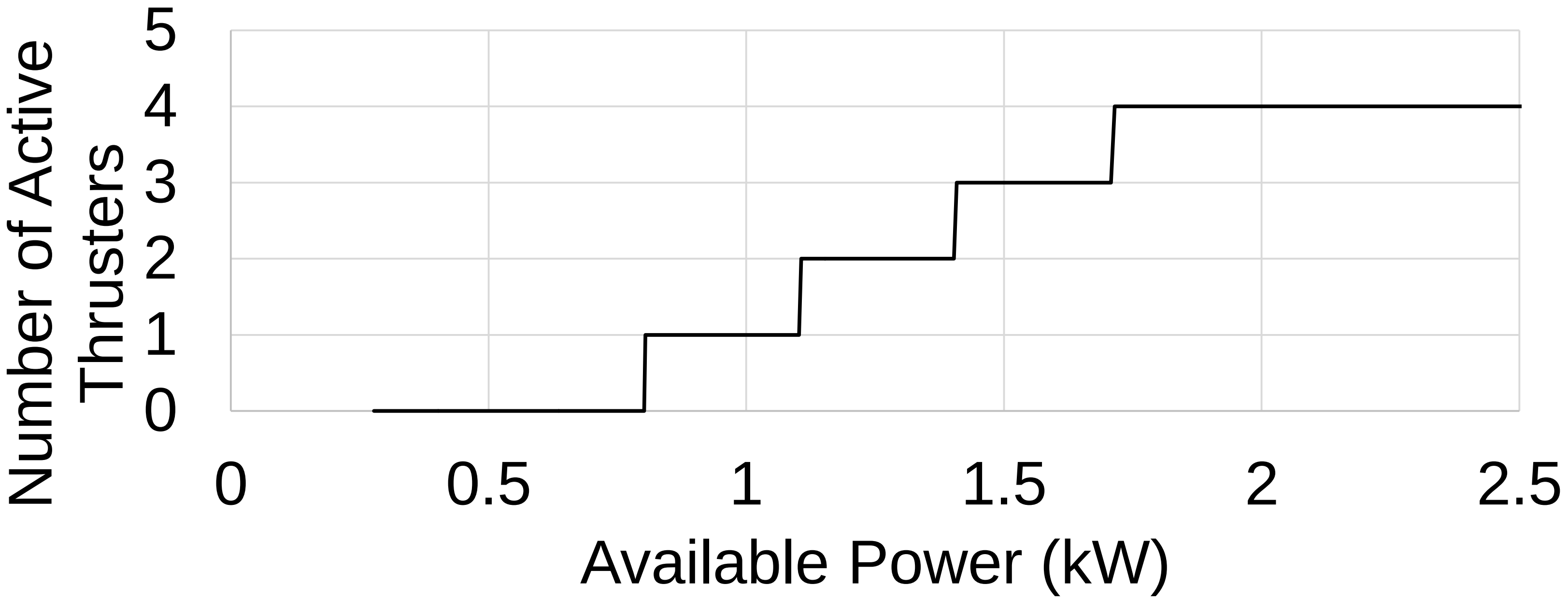
- Accurate design of low-thrust trajectories requires good modeling of spacecraft propulsion and power. Power drops as solar distance increases and propulsion trends nonlinearly with power.
- If a gradient-based optimizer is to be used, the propulsion and power models must be continuously differentiable.
- It is reasonable to model the performance of a low-thrust system as:

$$T = a_T P_{eff}^4 + b_T P_{eff}^3 + c_T P_{eff}^2 + d_T P_{eff} + e_T$$
$$\dot{m}_{max} = a_F P_{eff}^4 + b_F P_{eff}^3 + c_F P_{eff}^2 + d_F P_{eff} + e_F$$

$$P_{min} \leq P \leq P_{max}$$

Spacecraft Propulsion: The problem of Thruster Switching

- But what happens if you have multiple thrusters and you want to switch them on and off, or if there is not enough power to turn on a thruster?
- Moving outside the bounds of the minimum and maximum power for the thruster causes a discontinuity in $T(P)$ and $\dot{m}(P)$



Thruster Switching, Continued

- Problem: Gradient-based optimizers require continuously differentiable models, but thruster switching is inherently discontinuous. This makes the solver less robust.
- A candidate solution: Heaviside defined a step function as taking half value at the point of transition. This can be approximated using the logistics function:

$$H\left(x\right) = \lim_{k\to\infty} \frac{1}{1+\exp\left(-2kx\right)}$$

- We can define whether a given thruster is on or off by:

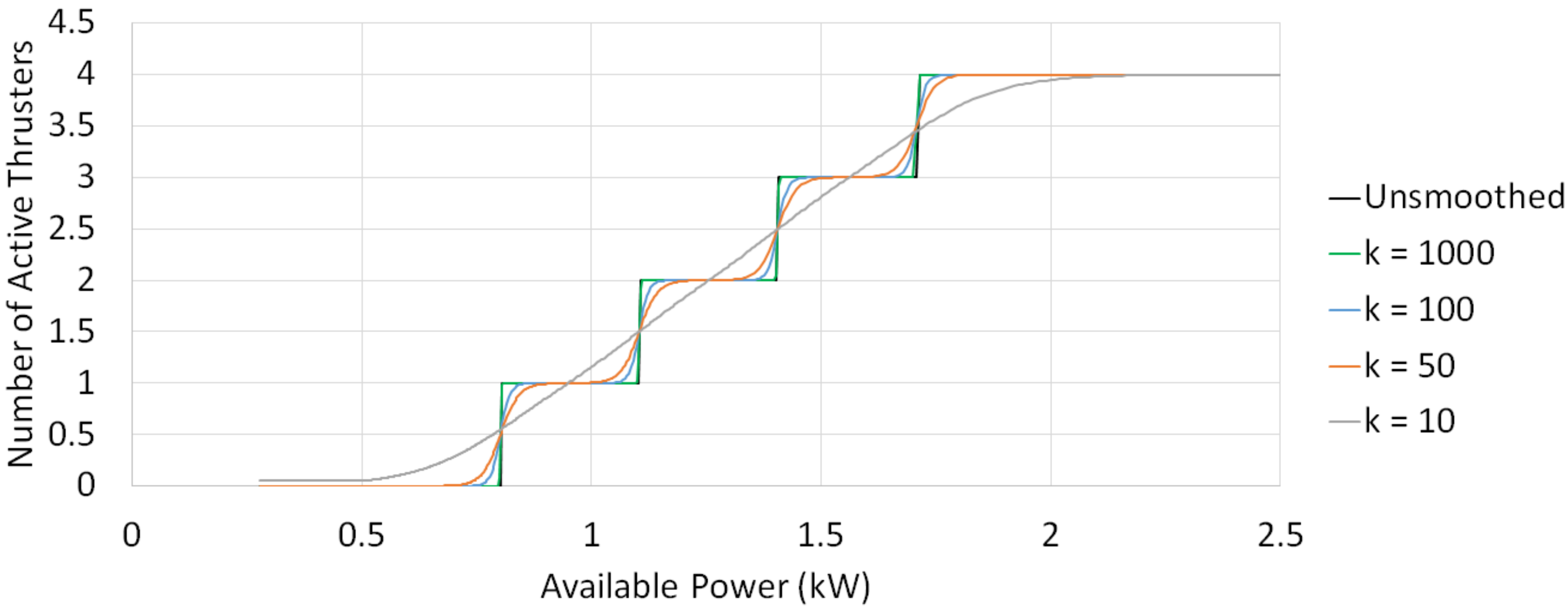
$$H_i\left(P\right) = \frac{1}{1+\exp\left(-2k\left(P-P_i^*\right)\right)}$$

where P_i^* is the power level at which a switch occurs. The total number of active thrusters may then be modeled as a continuous function,

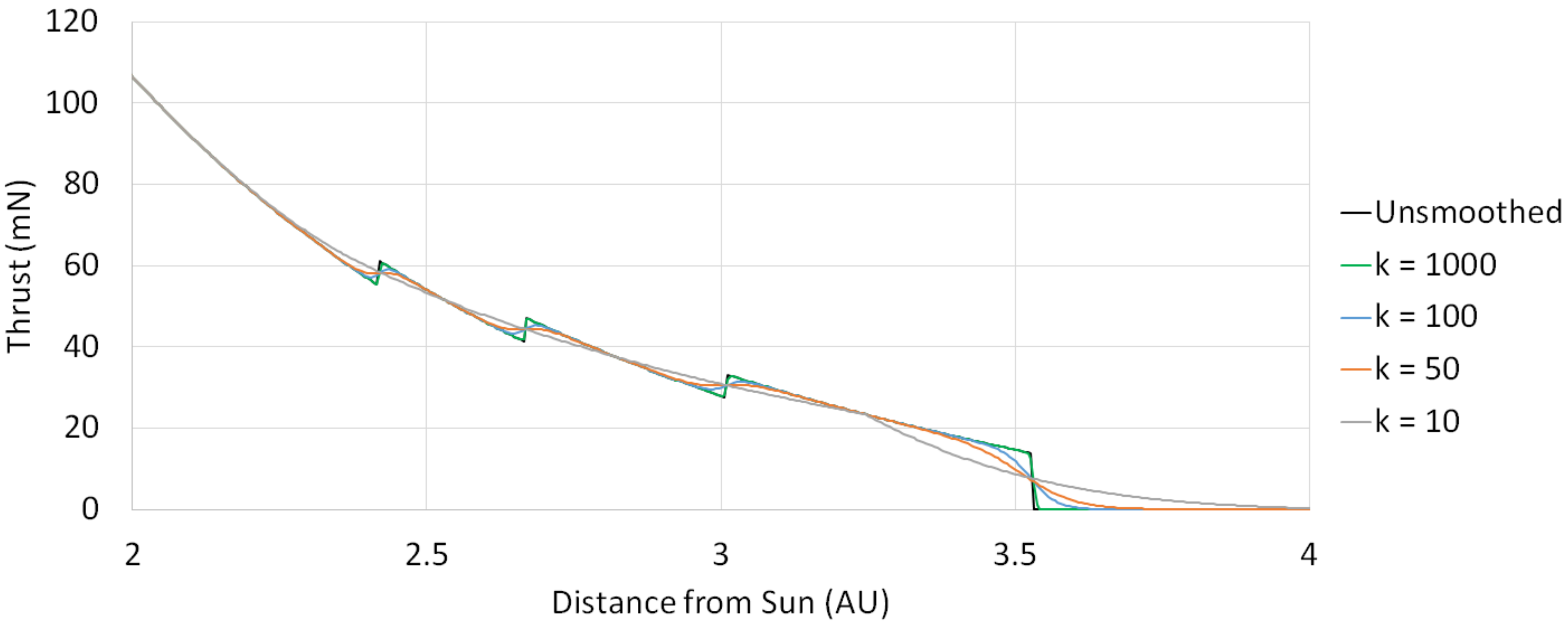
$$N_{active} = \sum_{i=1}^{N} H_i\left(P\right)$$

And the power per thruster as $P_{effective} = P/N_{active}$

Spacecraft Propulsion with Heaviside Smoother – Number of Thrusters



Spacecraft Propulsion with Heaviside Smoother – Available Thrust



GLOBAL SEARCH

Inner-Loop Solver: Nonlinear Programming (NLP)

Minimize $f(\boldsymbol{x})$

Subject to:

$$\boldsymbol{x}_{lb} \leq \boldsymbol{x} \leq \boldsymbol{x}_{ub}$$
$$\boldsymbol{c}(\boldsymbol{x}) \leq \boldsymbol{0}$$
$$\boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{0}$$

where:

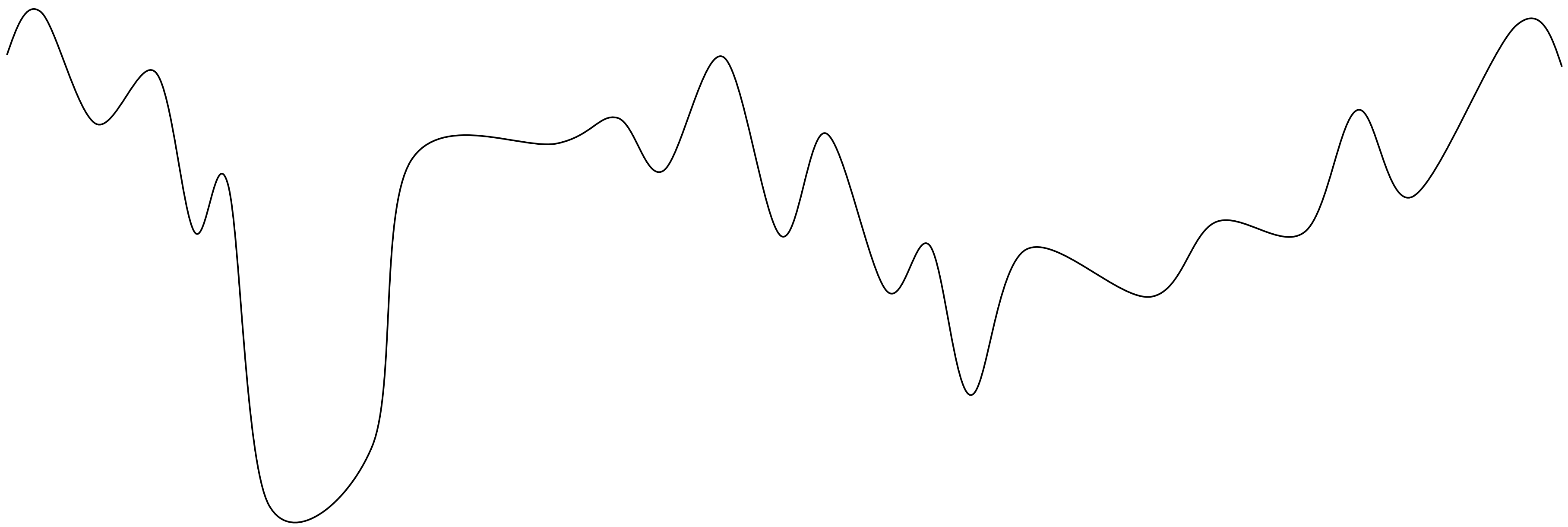
$\boldsymbol{x}_{lb}$, $\boldsymbol{x}_{ub}$ are lower and upper bounds on the decision variables

$\boldsymbol{c}(\boldsymbol{x})$ is a vector of nonlinear constraints

$\boldsymbol{A}\boldsymbol{x}$ is a vector of linear constraints

- There are several third party solvers that do this (SNOPT, IPOPT, fmincon, vf13AD)
- But all of these methods require an initial guess…

Inner-Loop Solver: Monotonic Basin Hopping (MBH)



Leary, 2000
Vasile, Minisci, and Locatelli, 2009
Yam, di Lorenzo, and Izzo, 2011
Englander (dissertation), 2013
Casioli *et al.*, 2013
Englander and Englander, 2014

Improved from standard MBH by:

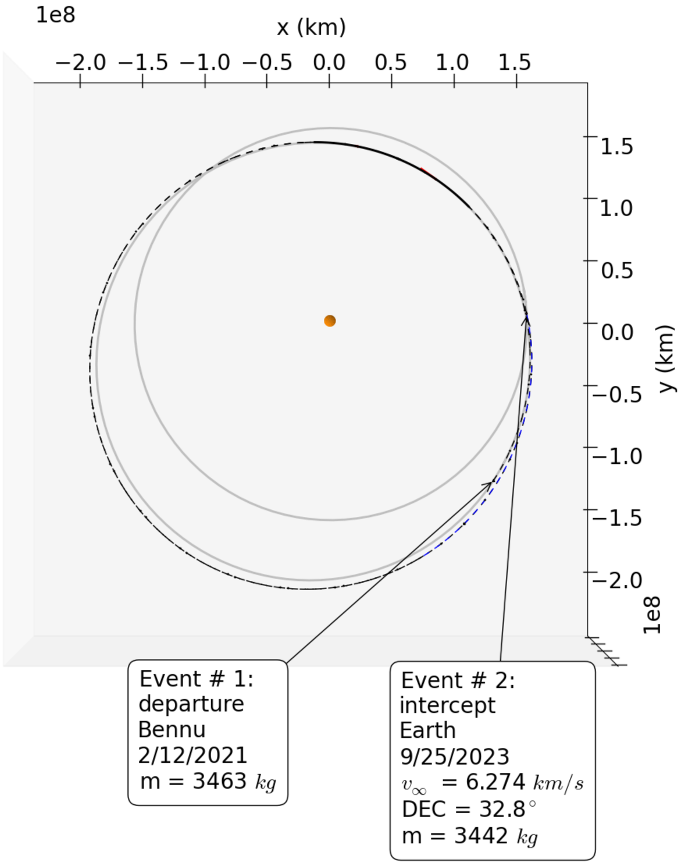
1. “Feasible point finder” aggregate penalty method
2. Non-uniform (Pareto) perturbation step
3. “Time-hop” operator (Casioli *et al.*)

EXAMPLES

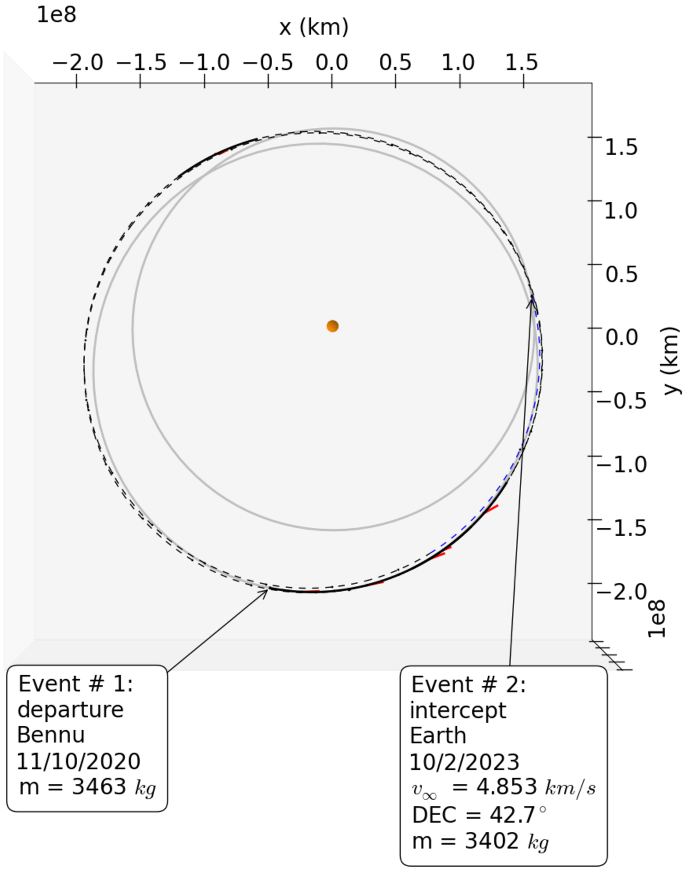
Example: LowSIRIS-REx

Option	Value
Launch window open date	1/1/2016
Launch window close date	1/1/2017
Flight time upper bound	7 years
Arrival condition at Bennu	rendezvous (match position and velocity)
Arrival condition at Earth	intercept with $v_{entry-relative} \leq 12.4 km/s$
Launch vehicle	Atlas V 411
Launch asymptote declination bounds	$[-28.5, 28.5]$ (Kennedy Space Center)
Post-launch coast duration	60 days
Pre-arrival coast duration	90 days
Solar array $P0$ at end of life, 1 AU	15 kW
Solar array model	$1/r^2$
Propulsion system	2 NEXT
Duty cycle	90%
Power margin	15%
Number of control steps per phase	40
MBH run time	3600 seconds

LowSIRIS-REx Trajectories



Unconstrained

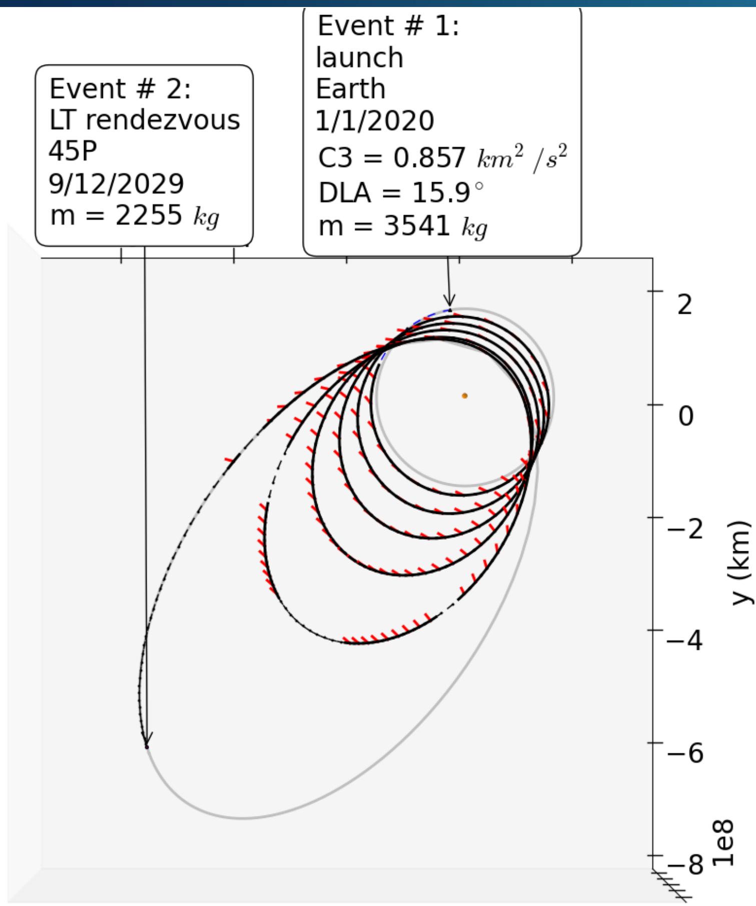


Constrained

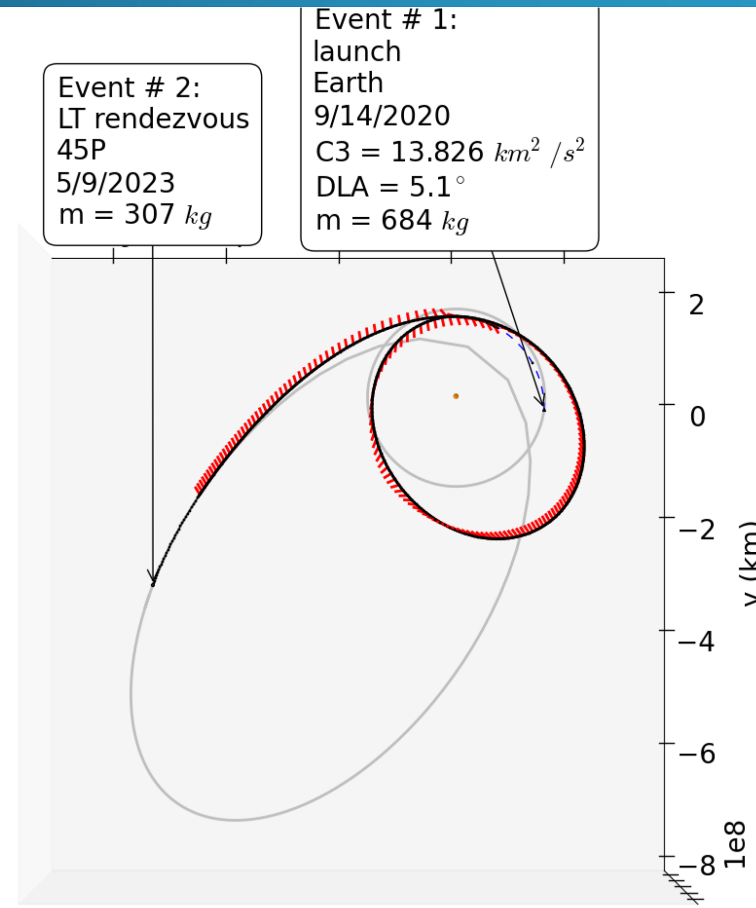
Example: Rendezvous with Comet 45P

Option	Value
Launch window open date	1/1/2020
Launch window close date	1/1/2026
Flight time upper bound	15 years
Arrival condition at 45P	rendezvous (match position and velocity)
Launch vehicle	Falcon 9 v1.1
Launch asymptote declination bounds	$[-28.5, 28.5]$ (Kennedy Space Center)
Post-launch coast duration	60 days
Solar array $P0$ at end of life, 1 AU	15 kW
Solar array model	$1/r^2$
Propulsion system	2 NEXT
Duty cycle	90%
Power margin	15%
Number of control steps per phase	200
MBH run time	3600 seconds

Example: 45P Rendezvous Trajectories



Unconstrained



Constrained

Conclusions

- Interplanetary space missions often must obey complex operational constraints and are governed by complicated force models.
- It is not always practical to search globally for unconstrained solutions and then apply constraints *a posteriori* – this can lead to **false positives** and **false negatives**.
- The ideal case is to pose the operational constraints in the global search problem.
- Two particularly annoying constraints and one force model addendum are presented in this work, along with a method for global search.
- This is certainly not the only way – and perhaps not even the best way.
- We encourage other authors to consider the problem of global search with operational constraints!

Thank You

EMTG is available open-source at

https://sourceforge.net/projects/emtg/